Matter-antimatter asymmetry generated at low temperatures

José Manuel Carmona¹, José Luis Cortés¹, Ashok Das², Jorge Gamboa³, and Fernando Méndez⁴

¹ Departamento de Física Teórica, Universidad de Zaragoza, 50009 Zaragoza, Spain
² Department of Physics and Astronomy, University of Rochester, NY 14627-0171, USA
³ Departamento de Física, Universidad de Santiago de Chile, Casilla 307, Santiago 2, Chile
⁴ INFN, Laboratorio Nazionale del Gran Sasso, SS, 17bis, 67010 Asergi (L’Aquila), Italy

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Abstract. We explore the possibility of having baryogenesis at temperatures below any dilution mechanism, such as inflation or sphaleron effects at the electroweak transition. We find that large infrared CPT violation (CPTV) effects are a necessary ingredient for such a baryogenesis. We give an explicit example of a theory supporting this scenario at the kinematical level in which such CPTV effects are suppressed at the level of CPT tests based on mass differences between particle and antiparticle.

In his seminal paper [1] Sakharov outlined three ingredients that are essential for an initially baryon-symmetric universe to dynamically evolve into one with a baryon asymmetry. These are: the presence of $B$ nonconserving interactions, violation of both $C$ and $CP$, and a departure from thermal equilibrium. It is clear that $B$ must be violated if the universe starts out as baryon symmetric and then generates a net baryon number $B$. Since the initial state with $B = 0$ is invariant under $C$ and $CP$, it will remain so, with the $B$-nonconserving reactions producing baryon and antibaryon excess at the same rate, unless both $C$ and $CP$ are violated. Finally, in thermal equilibrium particle phase space distributions are given by $f(p) = \exp((E + \mu)/T) \pm 1$, and their densities by $n = \int d^3p f(p)/(2\pi)^3$. Here $E$, $\mu$ denote respectively the energy and the chemical potential of the particle. In chemical equilibrium the entropy is maximal when the chemical potential of the particle $\mu$ is equal to $\mu = 0$. CPT invariance ensures that $n_b = n_\bar{b}$ so that $E = \sqrt{p^2 + m^2}$ is the same for baryons and antibaryons. As a result $n_b = n_\bar{b}$ unless there is a departure from thermal equilibrium.

This departure from thermal equilibrium is implemented through specific mechanisms. In GUT models the origin of the baryon asymmetry of the universe is explained through the existence of massive bosons whose interactions violate $B$ conservation. If their masses are sufficiently large ($> 10^{17}$ GeV), they will decay out of thermal equilibrium, producing a net baryon number. However, if inflation is produced after this baryogenesis, it would wash out the small asymmetry generated, and one should find alternative mechanisms to generate this.

Alternatives to GUT-baryogenesis have also been studied in recent years [2] through electroweak-baryogenesis, leptogenesis and Affleck-Dine baryogenesis, but it seems difficult to generate asymmetry of the right order of magnitude in any model consistent with present phenomenology.

In view of these difficulties one could ask whether a mechanism of baryogenesis is possible at low temperatures, after all the dilution effects have occurred. One faces an immediate problem, namely, there is no further possible departure from thermodynamical equilibrium with $B$-violating processes after the electroweak transition, so that the third of Sakharov’s criteria can no longer be satisfied. However, re-examining this condition, it is evident that a possibility of baryogenesis in thermal equilibrium is possible if one allows for a violation of CPT [3–5] symmetry.

CPT invariance is a fundamental symmetry of quantum field theory (QFT), which is the framework of present microscopic theories, in particular the SM. The difficulties in formulating a consistent QFT containing gravitation has led to questions about some of the underlying assumptions of QFT. For example, recent developments in quantum gravity [6] suggest that Lorentz invariance may not be an exact symmetry at high energies. CPT conservation is also questioned within such contexts [7]. Spontaneous violation of CPT and Lorentz symmetry can also arise in string theories [8]. Recently CPT violation has also been considered in connection with neutrino physics [9].

In summary, the possibility of CPT violation is being considered quite extensively in recent years. One should, of course, note the most stringent limits on CPT violation coming from kaon systems, $(M_{K^0} - M_{\bar{K}^0})/(M_{K^0} + M_{\bar{K}^0}) < 10^{-19}$, as well as from the leptonic sector, $(M_{e^+} - M_{e^-})/(M_{e^+} + M_{e^-}) < 4 \times 10^{-8}$.

In other words, any CPT violating effect must necessarily be tiny. We are here interested in the corrections that such effects would produce in the calculation of the relative matter-antimatter densities $(n_b - n_\bar{b})$ in thermal equilibrium, possibly making $n_b \neq n_\bar{b}$ in the presence of
B-violating interactions, namely, with zero chemical potential. Since the densities will depend on temperature, the correction will be temperature dependent. Therefore, we can parameterize it by a parameter with dimensions of energy in natural units. If this parameter, \( \kappa \), satisfies \( \kappa \ll T \) and we assume that the particle mass \( (m) \) is much smaller than the temperature so that we can neglect any \( m/T \) dependence, then

\[
\frac{n_b - n_q}{n_b} = \frac{n_b}{n_b} - 1 \sim \frac{\kappa}{T}. \tag{1}
\]

We will call this an infrared (IR) effect of CPT violation. The other possibility is that the dimensionful parameter \( M \) in this case, satisfies \( M \gg T \). In fact this case seems more natural from the point of view of high-energy quantum gravity effect, where \( M \) is a high-energy scale of the order of the Planck mass, \( 10^{19} \text{GeV}. \) The expected correction is then of the form

\[
\frac{n_b - n_q}{n_b} - 1 \sim \frac{T}{M}, \tag{2}
\]

and we will call it an ultraviolet (UV) correction.

There have been a few attempts to relate CPT violation with the generation of a matter-antimatter asymmetry \([4, 10]\). However, up to now this has been investigated within the context of a CPTV of the ultraviolet type, which in the framework of effective field theories means that it is generated by terms in the effective Lagrangian that are at least effectively conserved this ratio remains constant. As long as the expansion is isentropic and the baryon number is at least effectively conserved this ratio remains constant.

Eq. (1) applied to the quark-antiquark asymmetry implies that a baryon asymmetry can be generated during the evolution of the universe even in thermal equilibrium \((\text{in the presence of } B\text{-nonconserving interactions})\). Prior to \(10^{-6} \text{sec} \) after the big-bang, quarks and antiquarks were in thermal equilibrium with photons, and \( n_q \approx n_{\bar{q}} \approx n_{\gamma} \), so that

\[
\frac{n_q - n_{\bar{q}}}{n_q} \approx \frac{n_B}{3n_{\gamma}} \approx g_s \frac{n_B}{3s}. \tag{3}
\]

Since \( g_s \approx 10^2 \) for \( T \gtrsim 1 \text{ GeV} \),

\[
\frac{n_B}{3s} \approx 10^{-11} \left( \frac{\kappa}{eV} \right) \left( \frac{\text{GeV}}{T} \right). \tag{4}
\]

If \( B\)-nonconserving interactions decouple below a temperature \( T_D \), the value of \( \kappa \) necessary to reproduce the observed baryon asymmetry is

\[
\frac{\kappa}{eV} \approx \frac{10}{3} \frac{T_D}{\text{GeV}}. \tag{5}
\]

The baryon to photon ratio \( \eta \equiv n_b/n_{\gamma} \) is estimated from direct measurements to be around \( 10^{-9} \), which agrees with the value needed for the primordial nucleosynthesis. The number of photons in the universe has not remained constant, but has increased at various epochs when particle species have annihilated (e.g. \( e^\pm \) pairs at \( T \approx 0.5 \text{ MeV} \)). However, as in standard cosmology \([14]\), we assume that there has not been significant entropy production during the expansion (adiabatic expansion), so that the entropy per comoving volume \((\propto sR^3)\) has remained constant. This is also the case for the baryon number per comoving volume \((\propto n_B R^3 \propto n_B/s)\) in the absence of \( B\)-nonconserving interactions (or if they occur very slowly). Since the entropy density is related to the density of photons through the effective number of degrees of freedom \( g_s \) at any temperature as \( s \approx g_s n_\gamma \), we have at present

\[
\frac{n_B}{s} \approx \frac{1}{7} \eta \approx 10^{-10}. \tag{6}
\]

In this case, satisfies

\[
M \gg T \sim 1 \text{ GeV}. \tag{7}
\]

At present the baryon density \( (n_B) \) is observed to be much larger than the antibaryon density \((n_b \gg n_{\bar{q}})\). Therefore one can use the approximation \( n_B = n_b - n_{\bar{q}} \approx n_b \).

Consequently, in order to generate a matter-antimatter asymmetry at lower temperatures, we have to assume an IR effect of CPT violation. This may not be very unnatural and we will discuss briefly an explicit example later where such an IR effect does arise. We note that an IR scale correlated to an UV scale also arises naturally in non-commutative field theories \([11]\), in large extra dimensions \([12]\), and in considerations on entropy bounds \([13]\). In the case of an infrared CPTV, however, an alternative is required to the natural mechanism of suppression of ultraviolet CPTV effects based on the assumption that the ultraviolet scale is much higher than the accessible energies in the laboratory. We will address in the last part of this letter the question whether the existence of infrared CPTV can be made compatible with present tests of CPT.

At present the baryon density \( (n_{\gamma}) \) is observed to be much larger than the antibaryon density \((n_b \gg n_{\bar{q}})\). Therefore one can use the approximation \( n_B = n_b - n_{\bar{q}} \approx n_b \).
CPT violating effects should also be adequately suppressed to have compatibility with present experimental results. The simplest tests of CPT invariance are the equality between the masses and lifetimes of a particle and its antiparticle. We will present shortly an explicit example of an extension of QFT with such a suppression at the kinematic level, which allows for a generation of matter-antimatter asymmetry independent of the mass difference of particle and antiparticle. There are other tests of CPT invariance which assume specific dynamic realizations, such as the standard model extension which is a proposal based on the idea of spontaneous Lorentz and CPT violation in an underlying Lorentz-covariant theory [7]. Clock-comparison tests, experiments with spin-polarized matter and other QED tests are able to put strong bounds on different coefficients of CPT violating terms.

It seems difficult to make these bounds compatible with the order of magnitude of the infrared scale necessary to the mechanism of baryogenesis under discussion. But since a dynamical theory incorporating such a mechanism goes beyond the conventional relativistic QFT it is natural to expect that the effects of our infrared scale will not be contained in an effective Lagrangian. This prevents us from putting bounds on this scale from CPT and Lorentz invariance tests as well as from giving specific predictions of other effects. The necessity to go beyond effective theories also prevents us to be able to get definite conclusions based on a comparison of the rate of the B-violating interactions at the origin of baryogenesis and the rate of expansion of the universe. Such a comparison would require an specific dynamical scheme.

We now describe an example [16] of an extension of QFT where one can explicitly work out the matter antimatter asymmetry arising from CPT violation independently of the details of the dynamics. The extension is based on a generalization of the canonical commutation relations. The simplest example one can consider is the theory of a free complex scalar noncommutative field defined by the Hamiltonian

\[ H = \frac{1}{2} \sum_{i=1}^{2} \int d^3x \left[ \pi_i^2 + (\nabla \phi_i)^2 + m^2 \phi_i^2 \right], \quad (7) \]

and commutators

\[ [\pi_i(\mathbf{x}), \pi_j(\mathbf{x}')] = \epsilon_{ij} B \delta(\mathbf{x} - \mathbf{x}'), \quad (8) \]
\[ [\phi_i(\mathbf{x}), \phi_j(\mathbf{x}')] = \epsilon_{ij} \theta \delta(\mathbf{x} - \mathbf{x}'), \quad (9) \]
\[ [\phi_i(\mathbf{x}), \pi_j(\mathbf{x}')] = i \delta_{ij} \delta(\mathbf{x} - \mathbf{x}'), \quad (10) \]

where \( B \) and \( \theta \) characterizing the deformation of the canonical commutation relations carry dimensions of energy and length respectively.

In [16] it has been shown that (7-10) lead to an anisotropic quantum field theory in the sense that the second quantized Hamiltonian can be written in the diagonal form as

\[ H = \int \frac{d^3p}{(2\pi)^3} \left[ E(p) \left( \frac{\alpha^2 p^2 + 1}{2} \right) + \bar{E}(p) \left( \frac{\beta^2 p^2 + 1}{2} \right) \right], \quad (11) \]

where \( E(p) \) and \( \bar{E}(p) \) are given by

\[ E(p) = \omega(p) \left[ \sqrt{1 + \frac{\lambda^2}{4}} - \frac{\lambda}{2} \right], \quad (12) \]
\[ \bar{E}(p) = \omega(p) \left[ \sqrt{1 + \frac{\lambda^2}{4}} + \frac{\lambda}{2} \right], \quad (13) \]

and

\[ \omega(p) = \sqrt{p^2 + m^2}, \quad \lambda = \frac{B}{\omega(p)} \pm \theta \omega(p). \quad (14) \]

Thus we see that the free theory of the noncommutative scalar field is a quantum field theory where the symmetry between particles and antiparticles is lost. This is, of course, a consequence of the violation of Lorentz invariance which is manifest in the Lagrangian description.

When the momentum of the particle \( p \) is such that \( B < \omega(p) < \theta^{-1} \) then one has \( E(p) \approx \bar{E}(p) \approx \omega(p) \) and one recovers the standard relativistic theory with a particle-antiparticle symmetry. This symmetry, however, is lost both in the high energy limit \( \omega(p) \sim \theta^{-1} \) and in the low energy limit \( \omega(p) \sim B \).

Let us next show that this simple theory gives an explicit realization of the asymmetry between particles and antiparticles due to CPT violation in the infrared. Let us consider a system of the two types of particles in thermodynamical equilibrium at temperature \( T \). The number of particles of each type in a volume \( V \) is given by (we have set \( \mu = \bar{\mu} = 0 \) in anticipation that the fully interacting theory would have “B violation”)

\[ n = 4\pi V \int_0^{\infty} \frac{p^2 dp}{e^{\frac{p}{T}} - 1}, \quad \bar{n} = 4\pi V \int_0^{\infty} \frac{\bar{p}^2 d\bar{p}}{e^{\frac{\bar{p}}{T}} - 1}. \quad (16) \]

If we consider a temperature \( T \) such that \( \theta T \ll B/T \ll m/T \ll 1 \), then one has a tiny asymmetry arising from (16) due to the infrared scale \( B \).

\[ \frac{n}{\bar{n}} - 1 \approx \alpha \frac{B}{T}. \quad (17) \]

where we have neglected higher order terms in an expansion in powers of \( B/T \) as well as corrections due to the ultraviolet scale \( \theta^{-1} \) and the mass. The coefficient of the linear term, \( \alpha \), has the value

\[ \alpha = \int_0^{\infty} \frac{e^{\frac{p}{T}} p^2 dp}{e^{\frac{p}{T}} - 1} - \int_0^{\infty} \frac{e^{\frac{\bar{p}}{T}} \bar{p}^2 d\bar{p}}{e^{\frac{\bar{p}}{T}} - 1} = \frac{\zeta(2)}{\zeta(3)} \approx 1. \quad (18) \]

The result in (17) can be compared with the expression (1) for the baryon asymmetry induced by CPTV in the infrared and leads to the identification

\[ \kappa = \alpha B \approx B. \quad (19) \]
This shows that a very simple extension of QFT has the necessary ingredients to generate a matter-antimatter asymmetry induced by CPTV. In order to have a realistic model one should consider a theory with fermionic excitations and one should go beyond the free theory and incorporate interactions violating baryon number.

We can also use this simple model to comment on the relation between this asymmetry and the mass difference between the particle and the antiparticle. It is not clear how to define the mass of a particle when Lorentz invariance is violated. One can consider the effect of the infrared scale \( B \) on the kinematic analysis of any process. If one considers processes where the number of particles minus antiparticles remains constant (i.e., if one neglects interactions violating the \( U(1) \) symmetry of the free theory of the complex field) then one can easily see from the expressions in (12)-(13) that the only kinematic effect of the noncommutative parameter \( B \) is to replace \( m^2 \) by \( m^2 + B^2/4 \). In this case, the only difference from the conventional relativistic kinematic analysis is a lower bound \( (B^2/4) \) on the mass squared, but there is no reflection of the CPTV of the free theory at the level of a mass difference between particles and antiparticles. In the presence of interactions violating the \( U(1) \) symmetry (which we have assumed implicitly), however, the theory will generate small mass differences of the order of \( g^2 B \) for a weak coupling \( g \) of such interactions. This illustrates how a particle-antiparticle asymmetry can be generated through CPTV independent of the mass difference between particles and antiparticles which is necessary in any attempt to ascribe matter-antimatter asymmetry of our Universe to CPTV because of the very stringent experimental limits on CPT.

In conclusion, the considerations of CPTV effects, which have started to be taken seriously in recent years, lead to the possibility to generate the matter-antimatter asymmetry at low temperatures. We find however considerable restrictions on the size of CPTV effects and the temperature at which \( B \)-nonconserving interactions stop being relevant. Suppression mechanisms of the CPTV, similar to the one we have presented in a simple example, will be needed at the level of the dynamics of these interactions to make it compatible with experimental tests of CPT and \( B \) violation. In this scenario, one can reformulate the criteria for the observable matter-antimatter asymmetry as (a) the presence of \( B \)-violating processes down to an energy scale much lower than what is commonly assumed in GUT models; (b) \( C \) and \( CP \) violation; (c) CPTV parametrized by an infrared scale \( \mu \) which is of the order of a few eV if \( B \)-nonconserving interactions extend down to temperatures below the nucleon mass, or of the order of a KeV if such processes decouple at \( T_D \sim 100 \text{ GeV} \) (see Eq. (6)). It is an open question to find a dynamical realization of such a scenario (necesarily beyond the effective field theory framework) leading to a new explanation of the dominance of matter in the Universe and hopefully to other testable effects allowing us to discriminate between this and other possible mechanisms of baryogenesis.

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